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Orthogonal Frequency Division Multiplexing(OFDM) System

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Abstract—LTE (Long Term Evolution) and WiMAX systems play the major role in present and future communication systems as they are capable to integrate several applications. These systems demand high data rates and high spectral efficiency to support multiple applications. For higher data rates Orthogonal Frequency Division Multiplexing (OFDM) system is a promising advanced communication technique because of its spectral efficiency, ISI and multipath fading free transmission capability compared with single carrier system. In OFDM system increase in peak power due to coherent addition of sub carriers. OFDM signal is basically the sum of various independently modulated sine waves, and its amplitude has an approximately Rayleigh distribution. Amplitude of OFDM signal shows strong fluctuations and the resulting high Peak-to-Average. In this paper focused on the Single Carrier system, Multi Carrier Modulation system, basics of OFDM system, OFDM advantages and OFDM disadvantages.

Keywords— Inter Carrier Interference (ICI), Inter Symbol Interference (ISI), Peak to Average Power Ratio

I.INTRODUCTION

The Orthogonality Frequency Division Multiplexing (OFDM) system is a Multicarrier Modulation (MCM) system in which all subcarriers are orthogonal. Serial to Parallel converter, Inverse Fast Fourier Transform, Cyclic Prefix, Fast Fourier Transform, and Parallel to Serial converter modules are all part of the OFDM System.

Single Carrier Communication system:

Consider Single Carrier Communication system with two-sided band width (B.W) B for communication and symbol time T is 1/B.B=B.W.Every T seconds, send one symbol =T= $\frac{1}{B}$.Symbol rate = $\frac{1}{T}$ = B.In Single carrier Communication system, one carrier will utilize the complete bandwidth.

$$S(t) = X(k) \qquad kT \le t \le (k+1)T$$
$$X(1) \qquad T \le t \le 2T$$
$$X(2) \qquad 2T \le t \le 3T$$

One symbol every T Secs = $\frac{1}{B}$.

Multi Carrier Communication system:

Consider Multi Carrier Communication system with two-sided band width B for communication and number of sub carriers are N .

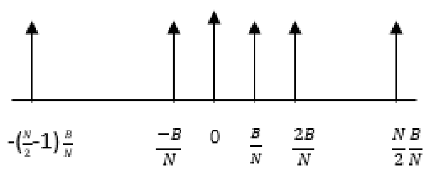

Fig. 1. MCM Sub Bands and Sub Carrier Spacing

The available B.W in a multi carrier communication system is divided into N equal sub bands, with each sub carrier spacing of B/N. The B.W of each sub carrier = B/N.Consider B.W B= 512 KHz and no of sub carriers N=512 (No of sub carriers is power of 2) etc. No of sub carriers N=512 Band width of each sub carrier = B/N = 512/512 KHz = 1 KHz.

In multi carrier system, consider i^{th} sub carrier and its corresponding carrier center frequency is :fi = $\frac{iB}{N}$

$-\left(\frac{N}{2}-1\right) \leq i \leq N/2$

The data transmitted on ith sub carrier with center frequency fi is Xi, while the data transmitted signal on ith sub carrier is $s_i(t)$.

$$s_i(t) = X_i e^{j2\pi f_i t} = X_i e^{j2\pi \frac{B}{N} t}$$

X_i = i^{th} Sub carrier of data transmitter and f_i = i^{th} Sub carrier of central frequency

There are N subcarriers, therefore there are N data streams in the system. X_i = i^{th} data stream and data transmitted on i^{th} sub carrier so the composite Multi Carrier transmit signal for N sub carriers is

$$s(t) = \sum_i si(t) \; s_i(t)$$

= i^{th} data stream modulated on i^{th} sub carrier s(t) = $\sum_i si(t) = \sum_i \mathrm{Xi} e^{j2\pi f i t} = \sum_i \mathrm{Xi} e^{\frac{j2\pi B}{N} t}$

Detection of Multi Carrier data (without noise):

$y(t) = s(t) = \sum_i \mathrm{Xi} e^{\frac{j2\pi B}{N} t}$

Let us consider de-modulator is coherent de-modulator and each stream is Coherently demodulated with corresponding sub carrier is

$$= B/N \int_0^{\frac{N}{B}} y(t)\,(e^{j2\pi f l t})^* dt \quad \text{fl = lth sub carrier frequency}$$

$$= B/N \int_0^{\frac{N}{B}} \left(\sum_i \mathrm{Xi} e^{\frac{j2\pi B}{N} t}\right)\left(e^{\frac{-j2\pi lB}{Nt}}\right) dt$$

$$= B/N \sum_i \int_0^{\frac{N}{B}} Xi e^{\frac{j2\pi(i-l)B}{N} t}\, dt$$

Assume i-l = k and

Fundamental period T0 = $\frac{1}{f0} = \frac{N}{B}$ f0 = $\frac{B}{N}$

$$= \frac{B}{N} \sum_i \int_0^{\frac{N}{B}} Xi e^{j2\pi k f0 t}\, dt$$

$$\int_0^{\frac{N}{B}} e^{j2\pi k f0 t}\, dt = 0 \qquad \text{if } i \neq l$$

$$= \frac{N}{B} \qquad \text{if } i = l$$

If i = l

$$\int_0^{\frac{N}{B}} 1\, dt = \frac{N}{B}$$

Except lth sub carrier all the remaining sub carriers are orthogonal with lth sub carrier.
After demodulation process with lth coherent carrier

$$= \frac{B}{N} Xl \frac{N}{B} + 0 = Xl$$

$$y(t) = s(t) = \sum_i \mathrm{Xi} e^{j2\pi f i t} = \sum_i \mathrm{Xi} e^{\frac{j2\pi B}{N} t}$$

For decoding the lth stream, demodulate coherently with $e^{j2\pi lB/Nt}$

$$f0 = \frac{B}{N} \qquad \text{Time period of integration} = \frac{1}{f0} = \frac{N}{B}$$

$$= \frac{B}{N} \int_0^{\frac{N}{B}} \left(Xl + \sum_{i \neq l} Xi e^{j2\pi(i-l)B/Nt}\right) dt$$

Time period of integration = $\frac{1}{f0} = \frac{N}{B}$

$$= \frac{B}{N} Xl \frac{N}{B} + \frac{B}{N} \sum_{i \neq l} Xi \int_0^{\frac{N}{B}} e^{\frac{j2\pi(i-l)B}{N} t}\, dt = \frac{B}{N} Xl \frac{N}{\mathbf{B}} = Xl$$

Xl = Transmitted Information symbol on lth sub carrier So, Xl is recovered by coherent demodulation with $e^{\frac{j2\pi lB}{N} t}$

For recovering N symbols corresponding to N sub carriers, coherently demodulate with N sub carriers corresponding to l = $-\left(\frac{N}{2} - 1\right)$ to $\frac{N}{2}$.

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This scheme is not completely OFDM. It is somewhat basis for orthogonal frequency division multiplexing scheme and is called as called as Multi Carrier Modulation (MCM). The window of time associated with detection of each multi carrier signal is $\frac{N}{B}$ .So, in MCM, transmitting N symbols on N sub carriers in time period is $\frac{N}{B}$. N symbols in time period $\frac{N}{B}$ then Symbol rate = $\frac{N}{N/B}$ = B. Overall symbol rate same in single carrier and multi carrier system.

Transmitter Schematic of MCM System:

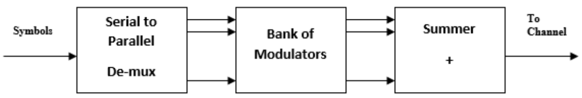

Fig. 2. Transmitter Schematic of MCM System

The Serial to Parallel converter converts the input serial data stream into N parallel data streams. The Bank of Modulators module is used to modulate ith data stream on to ith sub carrier. The summer module is used to combine all of the subcarriers into a single composite signal.

Receiver Schematic of MCM System:

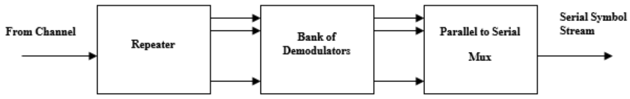

Fig. 3. Receiver Schematic of MCM System

For coherent demodulation with $(e^{j2\pi flt})*$ in MCM receiver, bank of demodulators module is using. For getting the single data stream from N parallel symbols, Parallel to serial converter is used.

Data rate:

In single carrier system data rate is 1 symbol in $\frac{1}{B}$.In MCM system data rate is N symbols in $\frac{N}{B}$.So, the data rate is same in both Single carrier system and Multi carrier system.

Example:

Consider Band width B = 2.048 MHz = 2048 KHz and typical coherence band width B_c is 200 to 300 KHz. In single carrier system, Bandwidth(B) ≥ Coherence Band width (B_c), so Inter Symbol Interference (ISI) happens in single carrier system. In Multi Carrier System Band width B = 2.048 MHz and Number of sub carriers are 1024.B = 2.048 MHz = 2048 KHz.Each sub carrier band width = $\frac{B}{N} = \frac{2048}{1024}$ KHz = 2 KHz. Typical coherence band width B_c is 200 to 300 KHz.

As a result, in a Multi Carrier Modulation system, the band width of each sub carrier (B/N) is less than or equal to the Coherence Band width (Bc), resulting in frequency flat fading for all sub carriers. As a result, no Inter Symbol Interference (ISI) occurs in the time domain in the MCM system. The development of a bank of N modulators and N demodulators at the transmitter and receiver is a disadvantage of the MCM system.

In 1971, Weinstein and Ebert, both engineers working in Bell Telephone laboratories, suggested "data transfer using frequency division multiplexing employing the Discrete Fourier Transform." The bank of N modulators at the transmitter and the bank of N demodulators at the receiver are eliminated using this proposed technique.

II. Orthogonal frequency Division Multiplexing (OFDM)

Consider the transmit signal of a MCM system, which has a B.W of B and thus a Nyquist sampling rate of B [4].

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Sampling time $T_S = \frac{1}{B}$
The composite signal in MCM system is

$$s(t) = \sum_i si(t) = \sum_i \text{Xiej2πfit} = \sum_i \text{Xiej2πB/Nt}$$

consider the u^{th} sample $t = uT_S = \frac{u}{B}$

$$s(uT_S) = x(u) = \sum_i \text{Xiej2π}\frac{B}{N} * \frac{u}{B} = \sum_i \text{Xiej2π}\frac{u}{N}$$

x (u) = samples of MCM signal
x(u) is IDFT (Inverse Discrete Fourier Transform) of information symbols X (0), X (1), X (2) --X(N-1)

$$s(t) = \sum_i \text{Xi}\, e^{j2\pi\frac{B}{N}t}$$

s(t) = MCM composite transmit signal

X_i = i^{th} stream

$T_S = \frac{1}{B}$

$uT_S = \frac{u}{B}$ = Time instant of the u^{th} sample

$$x(u) = s(uT_S) = s(u/B) = \sum_i \text{Xi}\, e^{j2\pi\frac{B}{N}*\frac{u}{B}} = \sum_i \text{Xi}\, e^{j2\pi\frac{u}{N}}$$

IDFT of transmission symbols $x(u) = \sum_i \text{Xiej2π}\frac{u}{N}$
IDFT can be utilised instead of Weinstein and Ebert's proposed bank of N modulators at transmitter and N demodulators at receiver for creating the composite transmit signal.
The "Inverse Fast Fourier Transform (IFFT)" method can be used to generate the MCM transmit signal [4].
The IFFT method of generating the MCM transmit signal have much lesser implementation complexity as compared with Bank of Modulators using in MCM system and IFFT with MCM is called as “Orthogonal Frequency Division Multiplexing (OFDM)” system. The FFT operation can be used at the receiver to retrieve the information symbols.

Transmitter Schematic of OFDM:

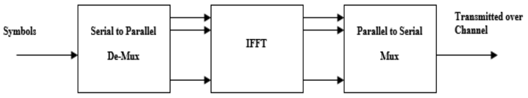

Fig. 4. Transmitter Schematic of OFDM

Receiver schematic of OFDM:

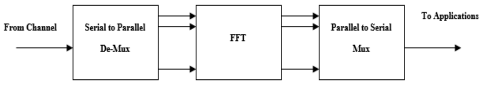

Fig. 5. Receiver schematic of OFDM

Cyclic Prefix:

The Transmitted symbols are X (0), X (1), X (2) …
The IFFT of X (0), X (1), X (2) … are x (0), x (1), x (2) …
x (0), x (1), x (2) … are samples of X (0), X (1), X (2) …
x {0}x{1} …x{N-1}x (0)x(1)x(2) ….x(N-1)
x {0}x{1} …x{N-1} is previous OFDM symbol.
x (0)x(1)x(2) ….x(N-1) is current OFDM symbol.
Frequency selective channel can be modelled as
h (0)h(1) ….h(L-1) (Multi Tap channel-L taps)
First output sample to current OFDM input symbol block is
y (0) = h (0)x(0) + h(1)x{N-1} + …. + h(L-1) x{N-L+1}
From above equation x{N-1}, x{N-2} +…x{N-L+1} are from previous OFDM symbol.
Initial samples are being subject to Inter Symbol Interference (ISI).

y (0) = h (0).x(0) + h(1).x{N-1} + . + h(L-1).x{N-L+1} [1]
The equation x{N-1} +….x{N-L+1) are from previous OFDM symbol and samples are being subject to ISI [8].
Now add x(N-L+1), x(N-L+2) …x(N-1) samples to x (0), x (1) ….x(N-1) then
y (0) = h (0).x(0) + h(1).x(N-1) +…. + h(N-1).x(N-L+1)…
y(n) = h(0)x(N-1) + h(1)x(N-2) +…+ h(L-1) x(N-L) [1]
From above equations observed that circular convolution.
Recovered samples [y (0) y (1) … y(N-1)] = [h (0) h (1) … h(L-1)] *[x (0) x (1) … x(N-1)]
Where x (0), x (1) …x(N-1) are N samples of the current OFDM symbol and are generated after the IFFT operation.
h (0), h (1) …h(L-1) is multitap channel models the Inter symbol Interference channel.
y (0), y (1) …y (N-1 are the recovered samples.
Y = h*x so Y(k) = H(k)X(k)
Above equation is possible by adding cyclic prefix.
Where Y(k) = N point DFT of y,
H(k)= N point DFT of h (after zero padding)
X(k) = modulated information symbols.
Y(k) is the output at k^{th} sub carrier(k) is the channel coefficient of k^{th} sub carrier and x(k) is the current symbol.
This is the flat fading channel across k^{th} sub carrier.
Hence, the frequency selective channel is converted into a group of narrow band flat fading channels across each sub carrier.[2]

Y (0) = H (0)X(0)
Y (1) = H (1)X(1)
Y(N-1) = H(N-1) X(N-1)

As a result, adding a Cyclic Prefix to OFDM effectively eliminates inter symbol interference, and OFDM coverts frequency selective channels into N parallel flat fading channels. The OFDM system divides a wide band channel with a band width of B into N parallel narrowband channels with a band width of B/N. Flat fading is applied to each narrowband channel.

OFDM Transmitter with Cyclic Prefix:

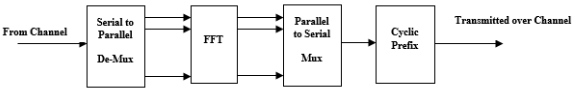

Fig. 6. OFDM Transmitter with Cyclic Prefix

OFDM Receiver with Cyclic Prefix:

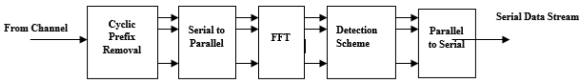

Fig. 7. OFDM Receiver with Cyclic Prefix

Consider channel noise N(k), Then
Y(k) = H(k)X(k) + N(k)
N(k) is the noise across the kth sub carrier.
The use of zero forcing across each sub carrier provides a basic X(k) detection strategy.
Detection of X(k) = 1/H(k). Y(k) = x(k)+N(k)/H(k)
Matched filter can also be employed for BPSK detection and Detection of X(k) = $(H(k))^*$. Y(k)=($|H(k)|^2$) X(k) + $(H(k))^*$. N(k).
Also, with MMSE receiver, detection of the k^{th} sub carrier in OFDM system is
Detection of $X_{MMSE}(k) = (H(k))^*)/((|(H(k))|^2) + \sigma n2$
Cyclic Prefix of the OFDM system is,
….x(N-2), x(N-1), x (0), x (1) …x (N-1)

L-1 is the cyclic prefix's minimum length. The cyclic prefix should be higher than the channel delay spread to avoid inter OFDM symbol interference. The information is unaffected by repeating the symbols. The main drawback of adding long cyclic prefix is loss in through put of the system.
Loss in efficiency = Cyclic Prefix/Total OFDM symbol length = $\frac{L-1}{N+L-1}$

$$\text{As } N\rightarrow\infty \lim_{n\to\infty}((L-1)/(N+L-1) \sim 0$$

As a result, a large number of sub carriers means lesser throughput loss and spectral efficiency loss approaches zero..
The OFDM symbol time N/B increases as the number of subcarriers N increases, which accounts for the ISI resulting from the delay spread [2].
The "Decoding Delay" of the system will increase as the number of sub carriers' N increases. As a result, there is a trade-off between increasing N and decreasing Decoding Delay.
Institutive framework to understand Cyclic Prefix (CP):

$$N_{cp} * T_s \geq T_d$$
$$N_{cp} \geq T_d/T_s = B*1/2B_c$$
$$N_{cp} \geq B/2B_c$$

No of symbols on Cyclic Prefix = N_{cp}
Bandwidth of the system= B
Coherence Bandwidth of the system= B_c

$$\text{For efficiency, } N \geq N_{cp} \geq B/2B_c$$
$$N \geq B/B_c$$
$$B_c \geq B/N$$

Coherence bandwidth (B_c) ≥ Bandwidth of each sub carrier(B/N)
As a result, each sub carrier is subjected to flat fading. As a result, an OFDM system that is properly built converts a frequency selective channel into a group of narrow band flat fading channels that span the carrier.

Effect of frequency off set in OFDM:

The available bandwidth is divided among a group of orthogonal overlapping sub carriers in OFDM. As a result of the loss of orthogonality among the sub carriers, the presence of a carrier frequency off set at the receiver can cause severe distortion in an OFDM system [7].

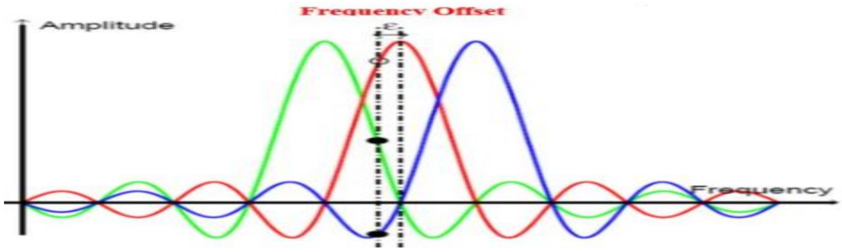

Fig. 8. Frequency off set in OFDM

Hence, presence of a carrier frequency off set, introduces Inter Carrier Interference (ICI) in OFDM system [5].
Characterize the effect of frequency off set on the performance of the OFDM system.
Consider a frequency off set Δf, such that
$\Delta f/(B/N) = \epsilon$
Δf = Frequency off set $\frac{B}{N}$ = sub carrier band width
ϵ = Normalized frequency off set
Given frequency off set ϵ, the base band received samples are given as

$$y(n) = \frac{1}{N}\sum_{k=-N/2}^{N/2} XkHke^{-j2\pi n(k+\epsilon)/N} + w_n$$

y(n) is nth received sample
X_k = Data Transmitted on k^{th} sub carrier
H_k = Channel coefficient across k^{th} sub carrier
ϵ = Normalized frequency off set w_n = noise

To verify the above equation, set $\epsilon=0$

$$y(n) = \frac{1}{N}\sum_{k=-N/2}^{N/2} XkHke^{-j2\pi n(k)/N} + w_n$$

Perform the FFT of y (0), y (1), y (2) …y(N-1) at the receiver.

Consider the l^{th} FFT coefficient Y_l, which corresponds to the symbol received on the l^{th} sub carrier. This is given as

$$Y_l = \frac{1}{N}\sum_n yne^{-j2\pi nl/N} + W_l$$

$$= \frac{1}{N}\sum_n \sum_{k=-N/2}^{N/2} XkHke^{j2\pi nk/N}e^{-j2\pi nl/N} + W_l$$

$$= \frac{1}{N}\sum_n \sum_{k=-N/2}^{N/2} XkHe^{j2\pi(k-l)n/N} + W_l$$

$$= X_lH_l + \frac{1}{N}\sum_n \sum_{k=-\frac{N}{2}\,\&\,k\neq l}^{N/2} XkHe^{j2\pi(k-l)n/N} + W_l$$

In the absence of frequency off set, $\epsilon=0$

$$Y_l = X_lH_l + W_l$$

Above equation is original OFDM relation to the l^{th} sub carrier.

Now consider the received symbols y_n in the presence of the carrier off set is

$$y(n) = 1/N\sum_{k=-N/2}^{N/2} XkHke^{-j2\pi n(k+\epsilon)/N} + w_n$$

After DFT at the receiver, the l^{th} coefficient Y_l, corresponding to symbol received on the l^{th} sub carrier is

$$Y_l = 1/N\sum_n yne^{-j2\pi nl/N} + W_l$$

$$= 1/N\sum_n \sum_{k=-N/2}^{N/2} XkHe^{j2\pi(k-l+\epsilon)n/N} + W_l$$

$$= 1/N\sum_n XlHl\, e^{j2\pi n\epsilon/N} + 1/N\sum_n \sum_{k=-\frac{N}{2}\&k\neq l}^{N/2} XkHe^{j2\pi(k-l+\epsilon)n/N} + W_l$$

$$\sum_{n=0}^{N-1} e^{j\theta n} = \sin(N\theta/2)/\sin(\theta/2)^* e^{j\emptyset}$$

$e^{j\emptyset}$ = Phase factor and does not effect the power

$$Y_l = \frac{1}{N} * XlHl\sin(\pi\epsilon)/\sin\left(\frac{\pi\epsilon}{N}\right) * e^{j\emptyset l} + \frac{1}{N}\sum_n \sum_{k=-\frac{N}{2}\&k\neq l}^{N/2} XkHe^{j2\pi(k-l+\epsilon)n/N} + W_l$$

Y_l = Desired signal Part + Inter Carrier Interference+ Gaussian Noise

Signal to Interference Noise Power:

Signal to Interference Noise Power is

SINR = Signal Power/ (Interference + Noise Power)

SINR = Signal Power/E{$|I_l|^2$} +σ_n^2)

Signal Power:

Signal Power = E{$|H_l|^2$} E{$|X_l|^2$} ($\sin(\pi\epsilon)/N\sin\left(\frac{\pi\epsilon}{N}\right)$)^2

For large no of sub carriers N,

$$\lim_{N\to\infty} \sin\left(\frac{\pi\epsilon}{N}\right) = \frac{\pi\epsilon}{N}$$

$$N\sin\left(\frac{\pi\epsilon}{N}\right) = N * \frac{\pi\epsilon}{N} = \pi\epsilon$$

Hence, the signal power for a large number of sub carriers N given as,

Signal Power = E{$|H_l|^2$} *P*(sin $\pi\epsilon$ / $\pi\epsilon$)^2

Signal Power = P$|H|^2$(sin $\pi\epsilon$ / $\pi\epsilon$)^2

Interference Power:

$$E\{|I_l|^2\} = E\{|H_l|^2\}\, E\{|X_l|^2\}\sum_{k=-\frac{N}{2}\&k\neq l}^{N/2}\left(\frac{\sin\pi\epsilon}{N\sin\frac{\pi(l-k+\epsilon)}{N}}\right)^2$$

Assume k-l = u and N→∞

$$E\{|I_l|^2\} = P|H|^2\sum_{u=\infty\&u\neq 0}^{\infty}\left(\frac{\sin\pi\epsilon}{N\sin\frac{\pi(u)}{N}}\right)^2 \quad u+\epsilon \cong u$$

$$E\{|I_l|^2\} = P|H|^2\sum_{u=\infty\&u\neq 0}^{\infty}\left(\frac{1}{N\sin\frac{\pi(u)}{N}}\right)^2$$

$$\sin\theta \geq 2\theta/\pi$$

$$\sin\pi u/N \geq 2\pi u/N/\pi = 2u/N$$

$$N\sin\frac{\pi u}{N} \geq 2u$$

$$E\{|I_l|^2\} \leq P|H|^2(\sin\pi\epsilon)^2 \sum_{u=\infty\&u\neq 0}^{\infty}\left(\frac{1}{2u}\right)^2 = P|H|^2(\sin\pi\epsilon)^2 * 2 * \sum_{u=1}^{\infty}\left(\frac{1}{2u}\right)^2$$

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$$\sum_{u=1}^{\infty}\left(\frac{1}{2u}\right)^2 = \pi^2/6$$

$$= \pi^2/12 * P|H|^2(\sin \pi\epsilon)^2 = 0.822\, P|H|^2(\sin \pi\epsilon)^2$$

Hence, SINR in the presence of carrier frequency off set ϵ is given by,
SINR (Approximation under large no of sub carriers N→∞) = $(P|H|^2(\sin \pi\epsilon / \pi\epsilon)^2) / (0.822\, P|H|^2(\sin \pi\epsilon)^2 + \sigma_n^2)$.

PAPR (Peak to Average Power Ratio):

PAPR in non-OFDM or single carrier System:

Consider a single carrier system, which is modulated with BPSK modulated symbols,
x (0) x (1) x (2) … +a -a +a…
Power in each symbol = Peak Power = a^2
Average Power = $E\{|x(k)|^2\} = a^2$
Therefore, in this single carrier system, both Peak and Average Power = a^2
PAPR = Peak Power/Average Power = 1 = 0dB
Hence, there is no considerable deviation from the mean power level [3].

PAPR in OFDM System:

Information symbols X (0), X (1), X (2) … +/- a, +/- a, +/- a
These information symbols are loaded onto sub carriers.

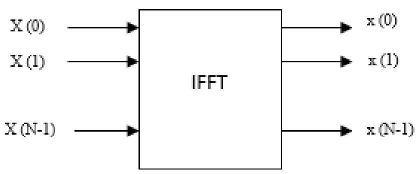

Fig. 9. IFFT Conversion

Hence, the transmitted samples are x (0), x (1) … x(n-1), which are IFFT samples of information symbols X (0), X (1) …X(N-1).
K^{th} IFFT sample $x(k) = 1/N\sum_{i=0}^{N-1} X(i)e^{j2\pi ki/N}$
Average Power = $E\{|x(k)|^2\}$

$= 1/N^2 \sum_{i=0}^{N-1} E\{square\ of\ |X(i)|\}\, E\{e^{\frac{j2\pi ki}{N}}\}^2 = 1/N^2 \sum_{i=0}^{N-1} E\{sqaure\ of\ (X\{i)\} = 1/N^2\sum_{i=0}^{N-1} a^2 = 1/N^{2*}a^2N = a^2/N$

Hence, the average power of transmission is a^2/N.
Peak Power = $x(0)^2$
$X(0) = \frac{1}{N}\sum_{i=0}^{N-1} X(i)e^{j2\pi(0)i/N} = \frac{1}{N}\sum_{i=0}^{N-1} X(i)$ X (0) = X (1) = X (2) =………= X(N-1) = +a
$X(0) = \frac{1}{N}\sum_{i=0}^{N-1} X(i) = \frac{1}{N}\sum_{i=0}^{N-1} a = a$
Peak Power = a^2
Hence Peak to Average Power Ratio is
PAPR = $a^2/a^2/N = N$

TABLE I. PAPR IN OFDM SYSTEM

No of Sub carriers(N)	64	128	256	512	1024
PAPR	64	128	256	512	1024
PAPR (in dB)	18.06	21.07	24.08	27.09	30.1

As a result, PAPR in an OFDM system might be significantly higher. Furthermore, the PAPR increases as N, or the number of subcarriers, increases. The IFFT operation is primarily responsible for the high PAPR in an OFDM system. The sum of data symbols over sub carriers can result in a signal with a high peak

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value [2]. The PAPR at the output of an OFDM system with 512 sub carriers and BPSK modulation, for example, can be as high as 27 dB.

I. *Effect of PAPR in OFDM system:*

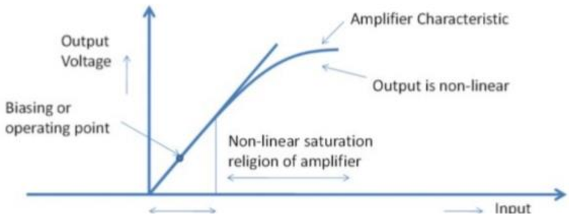

Fig. 10. Effect of PAPR on OFDM system

Peak voltage value variations are particularly large in an OFDM system as compared to the average value, and the output voltage value is moved outside of the linear range. Inter-Carrier-Interference (ICI) will arise in the OFDM system because to the high PAPR. Because of the high PAPR in the OFDM system, the RF power amplifier efficiency would suffer. The data transmission range in multi carrier systems will be reduced and spectral growth will be reduced if the transmitted power peak value is reduced for regulatory standards and application limits. If the system has a high PAPR value, the power amplifier will operate in the non-linear zone, which will reduce the battery life time. As a result, the benefits of multicarrier communication will be rolled out with a high PAPR value.
The signal is transmitted through the power amplifier in a practical OFDM system, but the peak strength of the power amplifier is limited. If the signal power exceeds the saturation point, the signal will be clipped. If clipping occurs in an OFDM system, the orthogonality property between sub carriers is destroyed, resulting in an increase in bit error rate. Non-linear power amplifiers were necessary in practically all communication systems for longer-range data transmission, but the power amplifier had to operate in a greater linear range for improved efficiency. As a result, efficiency and linearity are mutually exclusive.
Because the signal amplitude is measured in single-carrier modulation, the operating point measurement is precise. The signal amplitude changes with the data symbols in multi-carrier modulation systems like OFDM. As a result, variations in operating point will be observed, resulting in system distortion. At the receiver, this distortion acts as noise, and the signal constellation rotates as a result of the phase conversion. Cross talk is caused by out-of-band distortion of the subcarriers, as well as the orthogonality between subcarriers.
To determine the distortion induced by non-linearity, the signal must be measured to demonstrate its susceptibility to non-linearity. The following aspects must be considered before selecting the appropriate PAPR reduction approach in a digital communication system. Increased transmit signal power, reduced PAPR capacity, increased Bit Error Rate at the receiver, data rate loss, computation complexity, and so on. The majority of approaches are ineffective for reducing PAPR with little coding, low complexity, no performance deterioration, and no symbol handshake between transmitter and receiver.

Complementary cumulative distribution function (CCDF):

The complementary cumulative distribution function (CCDF) curve is used to define a digitally modulated signal's peak power statistics. In the temporal and frequency domains, most digitally modulated signals appear to be noise. Because of the non-linearity, statistical signal measurements are the best way to characterise the signals. The CCDF curves are quite beneficial when it comes to determining design parameters in digital communications systems. The CCDF curve shows how much time the waveform spends at or above a specific power level. The likelihood of a power level is calculated by measuring the amount of time the signal spends at or above that power level.
The Cumulative Distribution Function is a very important and commonly used function for determining the performance of a specific PAPR reduction strategy. CCDF curves are commonly used instead of CDF curves, and they describe the chance that the PAPR of a data block would exceed a specific threshold value . The real and imaginary values of the signal become Gaussian distribution functions for a large number of sub carriers N in a multi carrier system, according to the definition of the central limit theorem, and the amplitude of the OFDM signal follows the Rayleigh distribution function. The cumulative distribution for a multi-carrier OFDM signal is defined as $F(z) = 1 - e^{-Z}$

The probability that the PAPR is lower than some threshold level can be described as
$P(PAPR \le z) = F(z) = (1 - e^{-Z})^N$
CCDF of PAPR of an N carrier OFDM is defined as: $P(PAPR > z) = 1 - P(PAPR \le z) = 1 - F(z)) = [1 - (1 - e^{-Z})^N]$ [3]

Algorithm for Calculation of PAPR:

- Step 1: Generate the message bits.
- Step 2: In order to generate code words message bits are encoded and modulated.
- Step 3: Apply the IFFT for the input symbols and generate samples
- Step 4: Compute the PAPR for the each OFDM sample
- PAPR= Peak Power/Average Power
- (i)Compute the maximum power of the sample
- (ii)Compute Mean or Average value of the sample
- (iii)Compute the PAPR of the sample
- (iv)Convert the PAPR into dB, i.e., $10*\log_{10}(PAPR)$
- (v)Repeat the above steps for all samples
- (vi) Find the CCDF (complementary cumulative distribution function) for PAPR values

Step 5: Draw the Semi-Log Graph between PAPR and CCDF(PAPR).

Because of the overlap feature, the spectrum may be efficiently utilized. Because OFDM divides the channel into narrowband flat fading sub-channels, it is more robust to frequency selective fading than single carrier systems [8]. Use a cyclic prefix to get rid of ISI [2] are the advantages of OFDM. The main drawback of OFDM is OFDM signals have a high peak to average power ratio (PAPR), power amplifiers with a higher dynamic range are required [4][5]. Compared to single carrier systems, OFDM is more sensitive to carrier frequency offset [2].

III. Multiple Input and Multiple Output-Orthogonal Frequency Division Multiplexing (MIMO-OFDM)

MIMO-OFDM is the combination of MIMO communication system with OFDM properties.
Analogous to OFDM, MIMO-OFDM transforms a frequency selective MIMO channels into multiple parallel flat fading MIMO channels [8]. So, MIMO-OFDM substantially simplifies base band receiver processing, by excluding the need of a complex MIMO equalizer [9].

Frequency selective MIMO channel:

Frequency selective SISO channel is demonstrated as an FIR channel filter [9].
System model is given as,
$y(k) = \sum_{l=0}^{L-1} h(l)x(k-l) + n(k)$
$y(k) = h(0)x(k) + h(1)x(k-1) + h(2)x(k-2) + \ldots + n(k)$
$h(1)x(k-1) + h(2)x(k-2) + \ldots$ will create Inter Symbol Interference (ISI).
Hence, a MIMO frequency selective channel can be modelled as a MIMO FIR filter [9].
Hence, $y(k) = \sum_{l=0}^{L-1} H(l)x(k-l) + n(k)$
y(k) = r x 1 received symbol vector
x(k-l) = k x 1 Transmitted symbol vector
H(l) = r x k channel matrix corresponding to lth tap
n(k) = r x 1 noise vector
writing this explicitly,
$y(k) = H(0)x(k) + H(1)x(k-1) + \ldots H(L-1)x(k-L+1) + n(k)$
x(k) is transmitted vector at time t
x(k-1) is transmitted vector at time t-1
x(k-L+1) is transmitted vector at time k-L+1
y(k) depends on the transmit vectors x(k), x(k-1) …x(k-L+1).so y(k) depends on previous transmit symbol vectors.

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Hence, this is an L-tap frequency selective MIMO channel [9].
Hence, in a MIMO frequency selective channel, ISI happens between present and previous transmitted symbol vectors [8].
Hence, in a MIMO-OFDM system, one needs to perform the IDFT or IFFT operation at each transmit antenna.

MIMO-OFDM Transmitter Schematic:

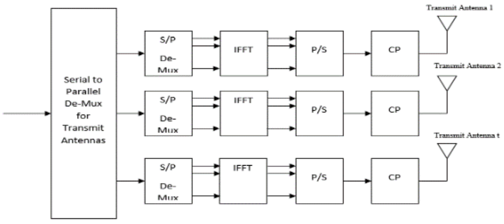

Fig. 11. MIMO-OFDM Transmitter Schematic

MIMO-OFDM Receiver Schematic:

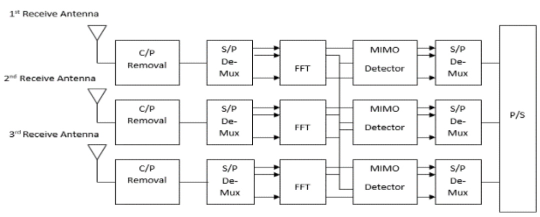

Fig. 12. MIMO-OFDM Receiver Schematic

Hence, MIMO frequency selective channel can be converted into a set of parallel flat fading MIMO channels.

$$Y(0) = H(0)X(0)$$
$$Y(1) = H(1)X(1) \ldots.$$
$$Y(N-1) = H(N-1)\,X(N-1)$$

Above are N parallel flat fading MIMO channels.
Each Y (0), Y (1) …Y(N-1) can be processed by a simple MIMO Zero Forcing (ZF) Receiver or MIMO MMSE Receiver for detection of vectors X (0), X (1) …, X(N-1).
MIMO Zero Forcing (ZF) Receiver:

$$\hat{X}(k) = (\tilde{H}(k))\dagger\bar{Y}(k) = (\tilde{H}(k)H\tilde{H}(k))\text{-}1\tilde{H}(k)\bar{Y}(k)$$

MIMO MMSE Receiver:

$$\hat{X}MMSE = Pd\,(Pd\tilde{H}(k)H\tilde{H}(k)+(\ \sigma n\ )2\ \tilde{H}(k)\bar{Y}(k)$$

Pd=Data Power
$\tilde{H}(0), \tilde{H}(1) \ldots \tilde{H}(N-1)$ are flat fading channel matrix across the sub carriers.
[$\widetilde{Hu}$,v(0), $\widetilde{Hu}$,v(1) … $\widetilde{Hu}$,v(N-1) =N Point FFT of [Hu, v (1), Hu, v (1) … Hu, v(L-1)] after zero padding [9].

V. Simulation Results

In this paper OFDM transmitter is implemented using different M-array QAM methods.
The following figure indicates the PAPR performance of conventional OFDM with 4-QAM modulation.

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TABLE II. PAPR PERFORMANCE IN CONVENTIONAL OFDM FOR 4-QAM

Modulation	OFDM 4-QAM	Single Carrier
No of Input Symbols	49152	49152
No of Symbol Blocks	192	192
IFFT Size	128	NA
PAPR (in dB)	9.6	1

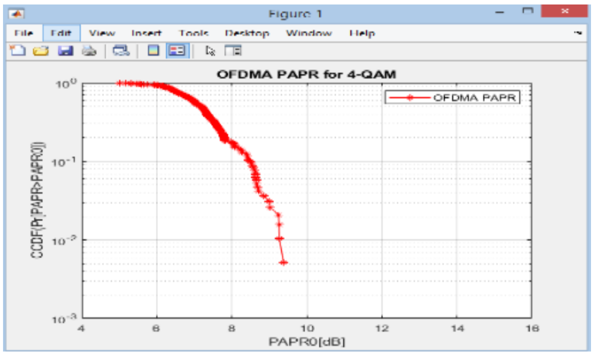

Fig. 13. PAPR Performance in conventional OFDM for 4-QAM

Figure-19 demonstrates conventional OFDM PAPR value is 9.6 dB at BER 10^{-2}.
The following figure indicates the PAPR performance of conventional OFDM with 8-QAM modulation.

TABLE III. PAPR PERFORMANCE IN CONVENTIONAL OFDM FOR 8-QAM

Modulation	OFDM 8-QAM	Single Carrier
No of Input Symbols	49152	49152
No of Symbol Blocks	128	128
IFFT Size	128	NA
PAPR (in dB)	9.9	0

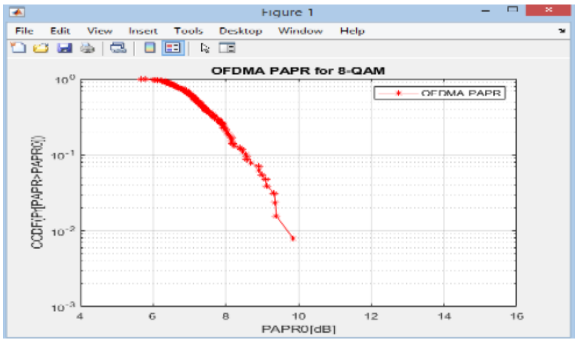

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Fig. 14. PAPR Performance in conventional OFDM for 8-QAM

Figure-20 demonstrates conventional OFDM PAPR value is 9.9 dB at BER 10^{-2}.
The following figure indicates the PAPR performance of conventional OFDM with 64-QAM modulation.

TABLE IV. PAPR PERFORMANCE IN CONVENTIONAL OFDM FOR 64-QAM

Modulation	OFDM 64-QAM	Single Carrier
No of Input Symbols	49152	49152
No of Symbol Blocks	64	64
IFFT Size	128	NA
PAPR (in dB)	10.3	0

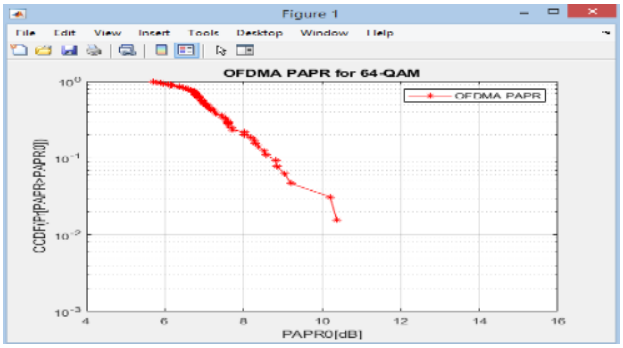

Fig. 15. PAPR Performance in conventional OFDM for 64-QAM

Figure-21 demonstrates conventional OFDM PAPR value is 10.3 dB at BER 10^{-2}.

IV.CONCLUSION

OFDM is a very promising technique for multicarrier transmission and has become one of the best choices for high – speed data transmission over a communication channel. This paper proves that OFDM is much suitable for multicarrier transmission than the single carrier transmission method. This paper provided the advantages of OFDM system, which are converting frequency selective channel to flat fading channel with the use of Cyclic Prefix (CP) and eliminating the Inter Symbol Interference (ISI). OFDM is mathematically efficient by using IFFT and FFT methods to employ the modulation and demodulation functions. It is more sensitive to carrier frequency offset than single carrier systems. The critical issue found in OFDM system is high Peak to Average Power Ration (PAPR).

REFERENCES

[1] A. A. A. Solyman, H. Attar, M. R. Khosravi, and B. Koyuncu, “MIMO-OFDM/OCDM low-complexity equalization under a doubly dispersive channel in wireless sensor networks,” Int. J. Distrib. Sens. Networks, vol. 16, no. 6, 2020, doi: 10.1177/1550147720912950.

[2] G. B. S. R. Naidu and V. M. Rao, “A Papr reduction of companded Sc-Fdma for 5g uplink communications,” Int. J. Innov. Technol. Explor. Eng., vol. 8, no. 7, pp. 135–139, 2019.

[3] C. S. Liu, “Solving singular convection–diffusion equation by exponentially-fitted trial functions and adjoint Trefftz test functions,” J. King Saud Univ. - Sci., vol. 30, no. 1, pp. 100–105, 2018, doi: 10.1016/j.jksus.2016.09.011.

[4] R. Giofre, P. Colantonio, and F. Giannini, "A Design Approach to Maximize the Efficiency vs. Linearity Trade-Off in Fixed and Modulated Load GaN Power Amplifiers," IEEE Access, vol. 6, pp. 9247–9255, 2018, doi: 10.1109/ACCESS.2018.2807479.

[5] H. H. Sneessens and O. Oguz, "Frequency-Selective Channel Advanced Transmitter and Receiver De- sign Space-Time Coding for Frequency Se- lective Channels," ScienceDirect, 2017.

[6] K. Singh, "PAPR Reduction with Amplitude Clipping & Filtering, SLM & PTS Techniques for MIMO-OFDM System: A Brief Review," Int. J. Recent Innov. Trends Comput. Commun., no. 0, pp. 1266–1270, 2017.

[7] B. K. Tiwari and C. K. Dwivedi, "Minimization of peak to average power ratio by selective mapping technique in OFDM environment," Int. J. Eng. Res. Technol., vol. 3, no. 4, pp. 5–11, 2016.

[8] M. Computing, N. Uppin, and V. Kaba, "Performance Analysis of Peak to Average Power Ratio Reduction Techniques," Int. J. Comput. Sci. Mob. Comput., vol. 4, no. 6, pp. 585–591, 2015.

[9] V. Sharma, "Various Techniques for PAPR Mitigation in OFDM System : A Survey," Int. J. Innov. Res. Electron. Commun., vol. 2, no. 2, pp. 1–6, 2015.

[10] B. Mohanty and B. Parida, "Reduction of peak to average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) system Bhawana Mohanty and 2 Bibhuti Parida," Quest Journals J. Electron. Commun. Eng. Res., vol. 2, no. 3, pp. 1–6, 2014.

[11] Z. Wang and S. Chen, "Reduction PAPR of OFDM Signals by Combining Grouped DCT Precoding with PTS," J. Signal Inf. Process., vol. 05, no. 04, pp. 135–142, 2014, doi: 10.4236/jsip.2014.54016.